CS513 HW3

Notice: you can use any known NP-hard problems for the polynomial reduction.

1. In a BoxTiling problem, there is a large Box R (in 3D) with width W, length L and height H, and several small boxes $r_1, r_2, \cdots, r_n$ where box r_i has width w_i, length ℓ_i and height h_i. The question is whether there is a subset of the small boxes that can be put inside the large box R with no gaps or overlaps. Show that BoxTiling is NP-hard.

2. Adam wants to ask if there is a simple path (i.e., not repeating vertices) in a graph G that goes through at least $1/3$ of the vertices. Show that this problem is NP-hard.

3. Consider a directed graph $G = (V, E)$, a number k, and a set of paths $P_1, P_2, \cdots P_m$ of G, is it possible to select at least k of the paths such that no two of the selected paths share any vertices? Show that this problem is NP-hard.

4. Given a finite set U of size n and a collection $A_1, A_2, \cdots A_m$ of subsets of U. You are alkso given numbers $c_1, c_2, \cdots c_m$. The question is that, does there exist a subset $X \subseteq U$ such that the cardinality of $X \cap A_i$ is equal to c_i? Prove that this problem is NP-complete.

   (Hint: you may use some variants of 3SAT: `https://en.wikipedia.org/wiki/Boolean_satisfiability_problem#3-satisfiability`)